
COMP 232 Mathematics for Computer Science
Winter 2014
Midterm Exam

Name: _____

Total Points:

ID: _____

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Instructions. This is a closed book exam. The only allowed tool is an ENCS approved calculator. Provide all answers in this booklet. Use pen, not pencil. Do not detach any pages from this exam!

- (3^{pts}_{ea.}) 1. We know that $\{\wedge, \neg\}$ forms a functionally complete set of operators, meaning that any other operator can be defined in terms of $\{\wedge, \neg\}$ only, for example

$$\begin{aligned}p \vee q &=_{\text{def}} \neg(\neg p \wedge \neg q) \\ p \rightarrow q &=_{\text{def}} \neg(p \wedge \neg q)\end{aligned}$$

The Shaffer stroke \uparrow is a binary operator that has the following truth table:

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

Show that the Shaffer stroke by itself is functionally complete, by defining in the space below, the following operators by using the Shaffer stroke only:

- (a) $\neg p =_{\text{def}} p \uparrow p$
- (b) $p \wedge q =_{\text{def}} (p \uparrow q) \uparrow (p \uparrow q)$
- (c) $p \vee q =_{\text{def}} (p \uparrow p) \uparrow (q \uparrow q)$

9 pts

9 pts

- (2_{ea.}pts) 2. For each of the following propositional sentences, state whether or not it is a tautology. You get +2 points for each correct answer, -2 points for each wrong answer, and 0 points for “don’t know.” However, the total for this question will not be less than 0.

10 pts

(a) $((p \vee q) \wedge (q \vee r)) \leftrightarrow (q \vee r)$

☒ Not tautology☐ Tautology☐ Don’t know!

(b) $(p \rightarrow q) \rightarrow (q \rightarrow p)$

☒ Tautology☐ Not a tautology☐ Don’t know!

(c) $((p \rightarrow q) \wedge (q \rightarrow p)) \leftrightarrow (p \leftrightarrow q)$

☒ Tautology☐ Not a tautology☐ Don’t know!

(d) $((p \rightarrow q) \wedge (\neg r \rightarrow \neg q)) \rightarrow (p \rightarrow (q \wedge r))$

☐ Tautology☒ Not a tautology☐ Don’t know!

(e) $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \rightarrow q) \rightarrow r)$

☒ Not a tautology☐ Tautology☐ Don’t know!

10 pts

(6pts_{ea.}) 3. Here you are to prove propositional equivalences using the laws in the handout.

12 pts

(a) In the table below, construct a proof of the equivalence

$$(r \vee p) \rightarrow (r \vee q) \equiv r \vee (p \rightarrow q)$$

Step	Law applied
$(r \vee p) \rightarrow (r \vee q) \equiv \neg(r \vee p) \vee (r \vee q)$	Implication
$\equiv ((\neg r) \wedge (\neg p)) \vee (r \vee q)$	de Morgan
$\equiv (((\neg r) \wedge (\neg p)) \vee r) \vee q$	Associativity
$\equiv (r \vee ((\neg r) \wedge (\neg p))) \vee q$	Commutativity
$\equiv ((r \vee (\neg r)) \wedge (r \vee (\neg p))) \vee q$	Distributivity
$\equiv ((r \vee (\neg p)) \wedge (r \vee (\neg r))) \vee q$	Commutativity
$\equiv ((r \vee (\neg p)) \wedge T) \vee q$	Excluded middle
$\equiv (r \vee (\neg p)) \vee q$	Identity
$\equiv r \vee ((\neg p) \vee q)$	Associativity
$\equiv r \vee (p \rightarrow q)$	Implication

(b) In the table below, construct a proof of the equivalence

$$\neg(p \rightarrow q) \rightarrow p \equiv \text{True}$$

Step	Law applied
$\neg(p \rightarrow q) \rightarrow p \equiv \neg\neg(p \rightarrow q) \vee p$	Implication
$\equiv (p \rightarrow q) \vee p$	Double negation
$\equiv (\neg p \vee q) \vee p$	Implication
$\equiv (q \vee \neg p) \vee p$	Commutativity
$\equiv q \vee (\neg p \vee p)$	Associativity
$\equiv q \vee \text{True}$	Excluded middle
$\equiv \text{True}$	Domination

12 pts

- (2pts_{ea.}) 4. Let the universe of discourse be \mathbb{Z}^+ , the set $\{1, 2, 3, \dots\}$ of positive integers. Let $P(x, y, z)$ denote the statement “ z is a multiple of $x+y$ ” or, equivalently, “There is a positive integer q , such that $z = (x+y)q$ ”

8 pts

What is the truth value of each of the following? You get +2 points for each correct answer, -2 points for each wrong answer, and 0 points for “don’t know.” However, the total for this question will not be less than 0.

(a) $\forall x \exists y \exists z P(x, y, z)$

☐ False

☒ True

☐ Don’t know!

(b) $\forall y \forall z \exists x P(x, y, z)$

☒ False

☐ True

☐ Don’t know!

(c) $\forall x \forall y \exists z P(x, y, z)$

☐ False

☒ True

☐ Don’t know!

(d) $\forall z \exists x \exists y P(x, y, z)$

☒ False

☐ True

☐ Don’t know!

8 pts

(4pts) 5. The negation of the statement $\forall x \neg \forall y \exists z (P(x, z) \wedge Q(z, y))$ is

- ☒ $\exists x \forall y \exists z (P(x, z) \wedge Q(z, y))$
☐ $\forall x \forall y \exists z (\neg P(x, z) \wedge \neg Q(z, y))$
☐ $\forall x \exists y \forall z (\neg P(x, z) \vee \neg Q(z, y))$
☐ $\forall x \exists y \forall z (\neg P(x, z) \wedge \neg Q(z, y))$
☐ $\exists x \exists y \forall z (P(x, z) \vee \neg Q(z, y))$

4 pts

(4pts) 6. The proposition $(p \leftrightarrow r) \rightarrow (q \leftrightarrow r)$ is equivalent to

- ☐ $((\neg p \vee r) \wedge (p \vee \neg r)) \vee \neg((\neg q \vee r) \wedge (q \vee \neg r))$
☐ $((\neg p \vee r) \wedge (p \vee \neg r)) \wedge ((\neg q \vee r) \wedge (q \vee \neg r))$
☒ $\neg((\neg p \vee r) \wedge (p \vee \neg r)) \vee ((\neg q \vee r) \wedge (q \vee \neg r))$
☐ $((\neg p \vee r) \wedge (p \vee \neg r)) \vee ((\neg q \vee r) \wedge (q \vee \neg r))$

4 pts

(4pts) 7. Which of the following statements is the contrapositive of the statement “You win the game if you know the rules but are not overconfident.”

- ☒ “If you lose the game then you don’t know the rules or you are overconfident.”
☐ “If you don’t know the rules and are overconfident then you win the game.”
☐ “If you don’t know the rules or are overconfident then you lose the game.”
☐ “A necessary condition that you know the rules or you are not overconfident is that you win the game.”
☐ “A sufficient condition that you win the game is that you know the rules or you are not overconfident.”

4 pts

12 pts

- (4pts) 8. Let $P(x, y)$ mean “ x loves y .” Which of the following expresses the proposition “Everybody loves somebody who doesn’t love anybody.”

4 pts

- ☒ $\forall x \exists y (P(x, y) \wedge \forall z (\neg P(y, z)))$
☐ $\forall x \exists y \forall z (P(x, y) \wedge \neg P(z, y))$
☐ $\forall x \forall y (P(x, y) \wedge \exists z (\neg P(y, z)))$
☐ $\forall x \exists y \exists z (P(x, y) \wedge \neg P(z, y))$

- (4pts) 9. Let the Universe of Discourse be \mathbb{Z} . Consider the assertion

$$\exists x (P(x) \wedge Q(x)) \equiv (\exists x (P(x))) \wedge (\exists x (Q(x)))$$

4 pts

Which of the following statements correctly describes the assertion?

- ☐ The assertion is false. As a counterexample, let $P(x)$ mean “ x is divisible by 6,” and $Q(x)$ mean “ x is divisible by 3.”
☒ The assertion is false. For a counterexample, let $P(x)$ mean “ $x < 0$,” and $Q(x)$ mean “ $x \geq 0$.”
☐ The assertion is false. For a counterexample, let $P(x)$ mean “ $x < 0$ is a square” and $Q(x)$ mean “ x is odd.”
☐ The assertion is true. The proof follows from the distributive laws for \wedge
☐ The assertion is true. To see why, let $P(x)$ mean “ x is divisible by 6,” and $Q(x)$ mean “ x is divisible by 3.” If $x = 6$, then x is divisible by both 3 and 6, so both side of the equivalence have the same truth value for this x .

—... End of Exam ...—

8 pts